Dividing & Rationalizing the Denominator Name: Key

Notes Date: ____________ Block: ______

Dividing Radicals

- **If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$**

Simplify.

1. $\sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \boxed{\frac{\sqrt{3}}{4}}$

2. $\frac{\sqrt{18x^5}}{\sqrt{2x^3}} = \sqrt{\frac{18x^5}{2x^3}} = \sqrt{9x^2} = \boxed{3x}$

3. $\frac{\sqrt[3]{162y^5}}{\sqrt[3]{3y^2}} = \sqrt[3]{\frac{162y^5}{3y^2}} = \sqrt[3]{54y^3} = \sqrt[3]{27 \cdot 2y^3}$

$\boxed{3y\sqrt[3]{2}}$

4. $\frac{\sqrt{50x^6}}{\sqrt{2x^4}} = \sqrt{\frac{50x^6}{2x^4}} = \sqrt{25x^2} = \boxed{5x}$

<u>What should you do when there is a square root in the denominator?</u>

For a radical expression to be "simplified"…

- No perfect square factors under radicals
- No radicals in denominators
- No denominators under radicals

To solve this simplification problem we are going to ***RATIONALIZE THE DENOMINATOR!***

Rationalize the denominator:

- Multiply the fraction by something equivalent to 1. (The same value to the top and bottom…)
- Goal: Create a perfect square/perfect nth factor in the denominator.

Simplify the radical expression completely.

5. $\frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{4}}$

$\frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2}}$

6. $\frac{6}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{6\sqrt{6}}{\sqrt{36}}$

$\frac{6\sqrt{6}}{6} = \boxed{\sqrt{6}}$

7. $\frac{3\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{10}}{\sqrt{4}}$

$\boxed{\frac{3\sqrt{10}}{2}}$

8. $\frac{\sqrt{x}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6x}}{\sqrt{36}} = \boxed{\frac{\sqrt{6x}}{6}}$

9. $\sqrt{\frac{9x}{2}} = \frac{\sqrt{9x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2x}}{\sqrt{4}}$

$\boxed{\frac{3\sqrt{2x}}{2}}$

10. $\frac{\sqrt{18x^2y}}{\sqrt{2y^3}} = \sqrt{\frac{18x^2y}{2y^3}} = \sqrt{\frac{9x^2}{y^2}}$

$\boxed{\frac{3x}{y}}$

11. $\frac{\sqrt[3]{7xy^2}}{\sqrt[3]{4x^2}} \cdot \frac{\sqrt[3]{2x}}{\sqrt[3]{2x}} = \frac{\sqrt[3]{14x^2y^2}}{\sqrt[3]{8x^3}}$

$\boxed{\frac{\sqrt[3]{14x^2y^2}}{2x}}$

12. $\sqrt{\frac{x}{8y}} = \frac{\sqrt{x}}{\sqrt{8y}} \cdot \frac{\sqrt{2y}}{\sqrt{2y}} = \frac{\sqrt{2xy}}{\sqrt{16y^2}}$

$\boxed{\frac{\sqrt{2xy}}{4y}}$

13. $\sqrt[4]{\frac{3a}{4b^2c}} = \frac{\sqrt[4]{3a}}{\sqrt[4]{4b^2c}} \cdot \frac{\sqrt[4]{4b^2c^3}}{\sqrt[4]{4b^2c^3}}$

$\frac{\sqrt[4]{12ab^2c^3}}{\sqrt[4]{16b^4c^4}} = \boxed{\frac{\sqrt[4]{12ab^2c^3}}{2bc}}$

$\frac{5}{2+\sqrt{3}} \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$ **Here our denominator is** $2+\sqrt{3}$ **so we want to multiply by its conjugate** $2-\sqrt{3}$.

$\frac{10-5\sqrt{3}}{4-\cancel{2\sqrt{3}}+\cancel{2\sqrt{3}}-\sqrt{9}} = \frac{10-5\sqrt{3}}{4-3} = \frac{10-5\sqrt{3}}{1} = \boxed{10-5\sqrt{3}}$

Simplify.

14. $\frac{3}{5-\sqrt{2}} \frac{(5+\sqrt{2})}{(5+\sqrt{2})} = \frac{15+3\sqrt{2}}{25-\cancel{5\sqrt{2}}+\cancel{5\sqrt{2}}-\sqrt{4}}$

$\frac{15+3\sqrt{2}}{25-2} = \boxed{\frac{15+3\sqrt{2}}{23}}$

15. $\frac{6}{10+\sqrt{2}} \frac{(10-\sqrt{2})}{(10-\sqrt{2})} = \frac{60-6\sqrt{2}}{100-\cancel{10\sqrt{2}}+\cancel{10\sqrt{2}}-\sqrt{4}}$

$\frac{60-6\sqrt{2}}{100-2} = \frac{60-6\sqrt{2}}{98} = \boxed{\frac{30-3\sqrt{2}}{49}}$

16. $\frac{9}{5-\sqrt{7}} \frac{(5+\sqrt{7})}{(5+\sqrt{7})} = \frac{45+9\sqrt{7}}{25-\cancel{5\sqrt{7}}+\cancel{5\sqrt{7}}-\sqrt{49}}$

$\frac{45+9\sqrt{7}}{25-7} = \frac{45+9\sqrt{7}}{18} = \boxed{\frac{5+\sqrt{7}}{2}}$

17. $\frac{\sqrt{3}}{\sqrt{3}-1} \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{\sqrt{9}+\sqrt{3}}{\sqrt{9}+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-1}$

$\frac{3+\sqrt{3}}{3-1} = \boxed{\frac{3+\sqrt{3}}{2}}$

18. $\frac{2\sqrt{7}}{\sqrt{3}-\sqrt{5}} \frac{(\sqrt{3}+\sqrt{5})}{(\sqrt{3}+\sqrt{5})}$

$\frac{2\sqrt{21}+2\sqrt{35}}{\sqrt{9}+\sqrt{15}-\sqrt{15}-\sqrt{25}} = \frac{2\sqrt{21}+2\sqrt{35}}{3-5}$

$\frac{2\sqrt{21}+2\sqrt{35}}{-2} = \boxed{-\sqrt{21}-\sqrt{35}}$

19. $\frac{4x}{3+\sqrt{6}} \frac{(3-\sqrt{6})}{(3-\sqrt{6})}$

$\frac{12x-4x\sqrt{6}}{9-\cancel{3\sqrt{6}}+\cancel{3\sqrt{6}}-\sqrt{36}} = \frac{12x-4x\sqrt{6}}{9-6}$

$\boxed{\frac{12x-4x\sqrt{6}}{3} = \frac{x(12-4\sqrt{6})}{3} = 4x-\frac{4}{3}x\sqrt{6}}$